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Noise Sources in Ultra-Low-Noise Synthesizer Design

This article, the second in a five-part series, focuses in on the synthesizer phase noise sources outside the synthesizer chip, such as noise induced in the voltage-controlled oscillator by power supplies and by the various loop-filter forms.

Phase noise is an interference source and a dynamic range limit in communications systems. The noise can cause interference both "in-channel" for systems with modulation terms close to the carrier, or further from the channel being used in what's often referred to as adjacent or alternate channel performance.

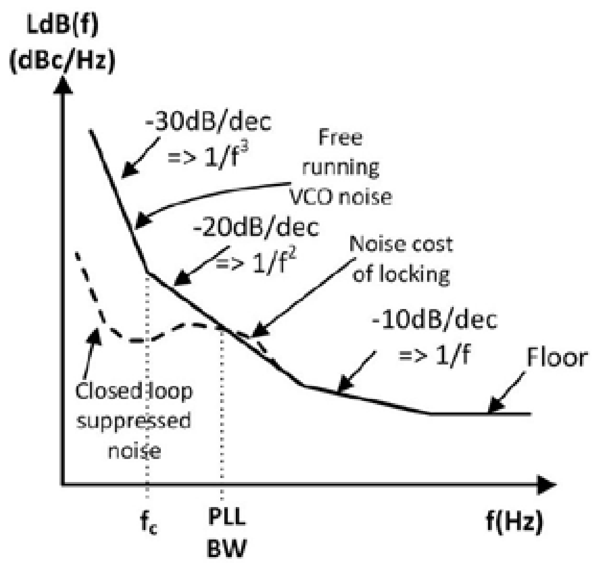
This is the second article in our low-noise synthesizer design series. A moderately longer version will be posted on the *Microwaves & RF* website, and a still more complete version will be posted on the Publications page at *www.longwingtech.com*. The first article (Ref. 1) covered basic design for functionality and stability. This second article, as well as the upcoming Part 3, will extend the basic methods to specifically cover designing for minimum phase noise. Here, Part 2 focuses on noise sources outside the synthesizer integrated circuit (IC), such as the voltage-controlled oscillator (VCO) and the various loop-filter forms.

The synthesizer IC noise and loop functions for shaping and combining all of the sources will be presented in the next article, revealing how modern synthesizer ICs with on-die VCOs can deliver performance that's often competitive and sometimes superior to synthesizers with low-noise discrete VCOs. The fourth article in this series will cover parts and CAD tools available to the low-noise synthesizer designer. The fifth and final article will bring this material together in the form of low-noise synthesizer examples.

FREQUENCY-DOMAIN DEFINITION OF PHASE NOISE

We speak of phase noise as the frequency-domain spectral-density noise surrounding the carrier of a frequency source, such as a VCO. It's described in units of decibels relative to carrier power per Hz, at an offset "f" from the carrier frequency *(Fig. 1)*. Refer to a source such as Ref. 2 for a more detailed definition.

In the shown VCO noise figure, the −30 dB/decade part of the slope below frequency f_c is flicker noise, which is predominantly caused by baseband flicker noise in the internal amplifier mixing up around the carrier. The −20 dB/decade slope is a basic indicator of Q in the oscillator loop. The −10 dB/decade part of the slope is typically only seen in very-high-Q oscillators. The floor is generated by the thermal noise raised by the gain and noise figure of the active device.



1. This is the phase noise of a free-running VCO and a phase-locked VCO. Here, the dB value will be referred to as $L_{dB}(f) = 10\log(L(f))$.

The phase noise as shown in *Figure 1* is single-sided. Consideration of total noise on both sides, the "spectral density of phase fluctuation" needed to covert phase noise to timing jitter, is discussed in the long version.

INDUCED NOISE IN THE VCO

Basic Induced Noise

Noise on the input (steering) voltage of the VCO will induce a sideband-to-carrier spectral component according to:

$$\frac{S}{C} = S_c = \frac{V_n K_{Hz}}{\sqrt{2}\, f} \quad (1)$$

We shall refer to this important relationship as the **"VCO Noise Modulation Function."** This equation is derived by small-signal FM theory. Here, K_{Hz} is the Hz/V steering function of the VCO and V_n is an rms modulating signal. If V_n is an rms spectral noise density, then this sideband-to-carrier becomes a phase noise density and is converted to dBc using 20log.

The noise-modulation function may also be used to give a phase-noise term due to noise on the supply through the power supply "frequency pushing" term K_{pHz}. Noise is modulated from this source to phase noise according to:

$$\frac{S}{C}(power\ supply) = S_{cp} = \frac{V_{np} K_{pHz}}{\sqrt{2}\, f} \quad (2)$$

For both input and power-supply noise, a flat noise spectral density will cause phase noise to decline at 20 dB/decade as frequency increases—similar to the most important part of a VCO noise curve. Over this frequency range, a constant (frequency flat) allowed noise density on steering and power nodes to be specified at a level that keeps this noise below the VCO free-running 20-dB/dec noise slope.

LOOP BW AND VCO K_{HZ} EFFECTS ON PHASE NOISE

The existence of the induced noise leads us to beware of noise on the VCO input, and to look for effects caused by such noise as a function of design choices such as loop BW and VCO gain. Well inside the loop bandwidth, such noises are suppressed by phase-locked-loop (PLL) closed-loop action. But at the loop BW, and for as much as a decade past it, we will have such noise sources acting unimpeded in inducing extra VCO noise. We know that the minimum noise we will have driving the VCO input will be given by the thermal noise of the zero resistor R_2. From the first article, we have this approximation for R_2:

$$R_2 = \frac{4\pi N \omega_n \zeta}{K_o I_{pd}} = \frac{2N\omega_n\zeta}{K_{Hz} I_{pd}} \cong \frac{N\omega_L}{K_{Hz} I_{pd}} \quad (3)$$

For the resistor thermal noise voltage, we have the standard equation (Ref. 3):

$$V_{nR2} = \sqrt{4kTR_2} \quad (4)$$

In the above, k is Boltzmann's constant (1.38E−23) and T is absolute temperature (290° at room temperature). Putting these two equations together with the induced noise relation (Equation 1), we find the minimum induced phase noise at the loop bandwidth f_L to be:

$$L_{MinInduced}(f_L) = \frac{4\pi kTN}{I_{pd}} \frac{K_{Hz}}{f_L} \quad (5)$$

In the locked loop, the noise at the loop bandwidth is a critical figure of merit. With a low-noise VCO, the induced noise of R_2 is usually an important component of that noise. This induced noise represents a minimum possible noise—even if there were no other PLL noise sources and the VCO were noise-free, this noise would still exist as the cost of locking the VCO.

LEESON'S EQUATION FOR VCO NOISE—AND ITS CONSEQUENCES

The Expanded Leeson Equation

A detailed expression for VCO phase noise is given below, which is an expansion of the famous equation first developed by Leeson (see long form for references) and further expanded upon by numerous authors. This expression is a linear power ratio. Therefore, in converting it to dB, we use 10log. We will use the variable "L(f)" for linear carrier-to-noise power ratio at offset "f." We will use "$L_{dB}(f)$" for the decibel variant.

In this expression, f is offset frequency; f_o is carrier frequency; f_c is the flicker noise corner; k is Boltzmann's constant; Q is *loaded* resonator Q; G is loop gain in compression (the reciprocal of VCO loop loss given by $1 - Q_L/Q_0$); F is noise factor in compression; K_{Hz} is VCO gain in Hz/volt; K_{pHz} is VCO power supply pulling in Hz per volt; V_{n1} is spectral noise density at offset "f" on the steering input; V_{n2} is spectral noise density at "f" on the power supply; and P_0 is the power dissipated within the loop (losses from all sources). More detailed information is provided in the full version at *www.longwingtech.com*.

$$L(f) \cong \frac{\left(\frac{f_o}{2Q}\right)^2 \frac{GFkT}{2P_0} f_c}{f^3} + \frac{\left(\frac{f_o}{2Q}\right)^2 \frac{GFkT}{2P_0} + \left(\frac{V_{n1}(f) K_{Hz}}{\sqrt{2}}\right)^2 + \left(\frac{V_{n2}(f) K_{pHz}}{\sqrt{2}}\right)^2}{f^2} + \frac{\frac{GFkT}{2P_0} f_c}{f} + \frac{GFkT}{2P_0} \quad (6)$$

Referring VCO Noise to Input

Leeson's equation and the earlier relationship on induced noise show that VCO noise at any frequency can be referred to the VCO input as a noise voltage that generates that same VCO noise on the output of a noiseless VCO. Given L(f) or $L_{dB}(f)$, we may write for input referred VCO noise V_{nvco}:

$$V_{nvco} = \frac{\sqrt{2}\, f\, 10^{\frac{LdB(f)}{20}}}{K_{Hz}} = \frac{\sqrt{2}\, f \sqrt{L(f)}}{K_{Hz}} \quad (7)$$

Noise on the power supply also generates noise on the VCO output, which will often require ultra-low-noise supplies for the VCO (discussed in Part 4 of the series). This noise may also be referred to the input for modeling purposes according to:

$$V_{npin} = V_{np}\frac{K_{pHz}}{K_{Hz}} \qquad (8)$$

In this equation, K_{pHz} is the VCO gain with respect to the power supply input in Hz/V and V_{np} is the spectral voltage noise density of the supply.

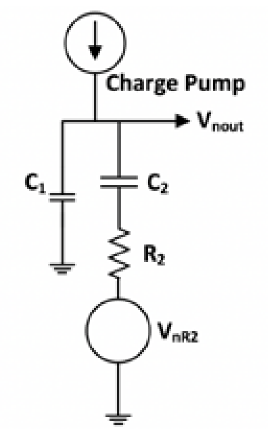
Examination of Leeson's equation shows the noise advantages of higher-frequency VCOs and dividing down in frequency, and of lower K_{Hz} for induced noise. These issues are discussed in more detail in the long version, as is conversion of phase noise to time jitter along with additional noise sources such as resistor excess noise.

FILTER NOISES AND LIMITS

Noise in the loop filter is for the most part suppressed by feedback well inside the loop bandwidth. However, around the loop bandwidth and for up to a decade or so past it, the noise of the filter may dominate the phase noise. As loop bandwidth is directly proportional to R_2 and other resistors in the passive loop filter generally scale with R_2, the filter noise may set limits on the bandwidth to be used in the PLL. The active loop filter has the advantage that the largest resistor noise will generally be from R_2, along with the disadvantage of op-amp and reference noises.

Second-Order Passive Filter Noise

The second-order passive filter has the form shown in *Fig. 2.*



2. Shown is the second-order loop filter with noise.

Analysis of the circuit yields:

$$|V_{noutR2}|^2 = \frac{4kTR_2}{\left(1+\frac{C_1}{C_2}\right)^2+(2\pi f R_2 C_1)^2} \qquad (9)$$

The filter impedance Z_f as seen by the charge pump is:

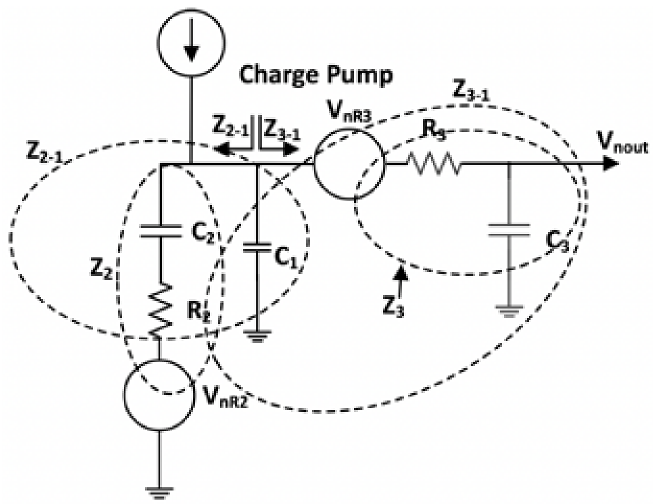
$$Z_f = \frac{sC_2R_2+1}{s(sC_1C_2R_2+C_1+C_2)} \qquad (10)$$

If the noise of the charge pump and dividers is modeled as a noise current I_{pn}, then:

$$|V_{ncp}|^2 = I_{pn}^2\,|Z_f|^2 = I_{pn}^2\,\frac{\omega^2 C_2^2 R_2^2+1}{\omega^4 R_2^2 C_1^2 C_2^2+\omega^2 (C_1+C_2)^2} \qquad (11)$$

The total filter noise squared is then the sum of these two noises. This form of loop filter represents the lowest possible noise, as it only has R_2 as an internal noise source.

Third-Order Passive Filter Noise



3. Depicted is the third-order passive loop filter with intermediate impedances defined for finding V_{nout}.

Figure 3 shows the third-order passive loop filter. In analyzing for V_{nout}, we define several intermediate impedances. We then apply voltage and current division to the different blocks. Going through this process (see long form for more details), we find the below set of equations:

$$Z_f(s) = \frac{1+sT_2}{s\,A_0(1+sT_1)(1+sT_3)} = \frac{1+sC_2R_2}{s\,(A_2 s^2+A_1 s+A_0)} \qquad (12)$$

$$T_2 = R_2C_2 \qquad (13)$$

$$T_3 = R_3C_3 \qquad (14)$$

$$T_{3-1} = \frac{C_1C_3}{C_1+C_3}R_3 \qquad (15)$$

The noise from R_2 at the output is:

$$V_{nout2} = \frac{4kTR_2}{\frac{C_1+C_3}{C_2}(sT_2+1)(sT_{3-1}+1)+(sT_3+1)} \qquad (16)$$

The noise from R_3 at the output is:

$$V_{nout3} = \frac{4kTR_3\,[C_1(sT_2+1)+C_2]}{(sT_3+1)[C_1(sT_2+1)+C_2]+C_3(sT_2+1)} \qquad (17)$$

And then finally:

Noise in the loop filter is for the most part suppressed by feedback well inside the loop bandwidth. However, around the loop bandwidth and for up to a decade or so past it, the noise of the filter may dominate the phase noise. As loop bandwidth is directly proportional to R_2 and other resistors in the passive loop filter generally scale with R_2, the filter noise may set limits on the bandwidth to be used in the PLL. The active loop filter has the advantage that the largest resistor noise will generally be from R_2, along with the disadvantage of op-amp and reference noises.

$$V_{nout} = \sqrt{V_{nout2}^2 + V_{nout3}^2} \qquad (18)$$

Compared to older integer-N designs, both R_2 and thus R_3 are smaller with the lower N value of modern fractional-N synthesizers that have higher phase-detector frequencies—and thus have lower noise. Oftentimes, R_3 is in the range of about R_2 to $3R_2$. With the properly designed third-order passive loop filter, we may thus approximate the noise voltage to be about 1.5 to 2 times that of the second-order filter.

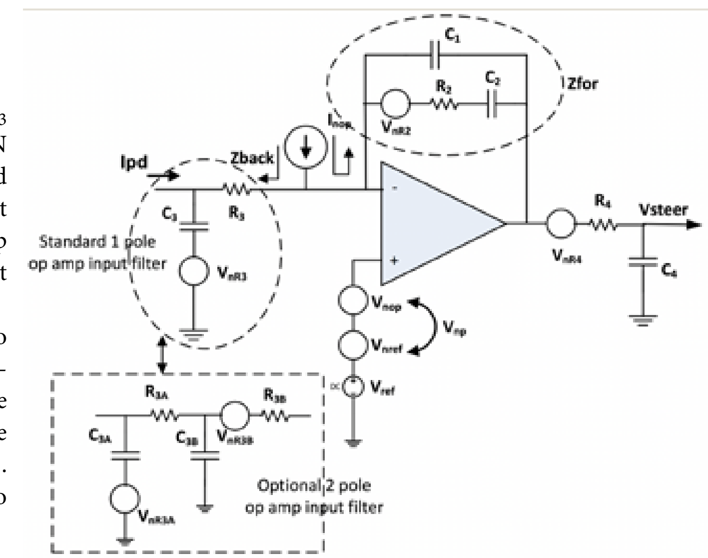
The lower R_2 and R_3 values are factors that contribute to lower total noise. Additional factors are the higher frequency to be divided down to the application frequency and the higher phase-detector frequency and loop bandwidth of the modern sigma-delta PLL (which will be presented in Part 3). For discrete VCOs, lower K_{Hz} allowed by higher voltage is also an essential factor (see long form for analytic details).

Third-Order Buffered Semi-Active Filter

A common active filter strategy is to break the third- or fourth-order passive filter up with an op-amp buffer in the middle that drives the final one or two RC stages, which shall be referred to here as "semi-active." Since it's not the most recommended active filter form, the analysis and design of this filter form is deferred to the full version of this article posted on the Longwing website.

Fourth-Order Active Filter Noise

The design methodology for the highly recommended "slow slew mode" fourth-order active filter was given in Part 1 of this series *(Fig. 4)*. This active filter architecture is designed to reduce the bandwidth and slew-rate requirements on the op amp, at which it's partially successful. The use of the inverting mode with R_2 and C_2 in the feedback path allows this form to provide higher tune voltages with low noise gain. Current flows through R_3 and then through R_2/C_2 to charge up the op-amp output to whatever voltage is needed. The input RC is intended to shield the op amp from the high slew rate and bandwidth of the charge-pump output.



4. The fourth-order active loop filter with option to become fifth-order. The noise sources to be analyzed are shown.

However, it may be advantageous to add an additional input RC stage. As pointed out by Banerjee, experimental evidence (Ref. 4, 5th ed., pp.371-372) indicates that when the op amp in an active loop filter is not fast enough, the 1/f noise of the PLL can sometimes increase by several dB. This is presumably due to pulse widening that allows more charge pump 1/f noise to go through.

The noise on the steering output of the active filter may be characterized as the sum of the noise powers from the plus input, the minus input, the forward impedance, and in the final RC stage.

The minus input noise is the thermal of R_3 gained up by:

$$V_{nopR3}^2 = 4kTR_3\left|\frac{Z_{for}}{Z_{back}}\right|^2 \qquad (19)$$

We can generate this over frequency using the next several relationships:

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$$Z_{back} = R_3 + \frac{1}{sC_3R_3} = \frac{sC_3R_3+1}{sC_3} \quad (20)$$

$$Z_{for} = \frac{1+sR_2C_2}{s(C_1+C_2+sR_2C_2C_1)} \quad (21)$$

Next, we consider the gained noise from the plus input of the op amp. This is the gained rms sum of the reference noise and the op amp's own noise, given by:

$$V_{nopp}^2 = V_{np}^2 \left|1 + \frac{Z_{for}}{Z_{back}}\right|^2 \quad (22)$$

The below magnitude squared functions are convenient to use:

$$|Z_{back}|^2 = R_3^2 + \frac{1}{\omega^2 C_3^2} = \frac{\omega^2 R_3^2 C_3^2 + 1}{\omega^2 C_3^2} \quad (23)$$

$$|Z_{for}|^2 = \frac{\omega^2 C_2^2 R_2^2 + 1}{\omega^4 R_2^2 C_1^2 C_2^2 + \omega^2 (C_1+C_2)^2} \quad (24)$$

The noise generated by R_2 is found by taking into account that the minus input of the op amp is a "virtual ground." Op-amp feedback holds it equal to the plus input. Therefore, noise from R_2 comes straight through to the output except in the case of being filtered within Z_{for} when $C_1 > 0$.

$$|V_{nopR2}|^2 = \frac{4kTR_2}{\left(1+\frac{C_1}{C_2}\right)^2 + (2\pi f R_2 C_1)^2} \quad (25)$$

The noise generated on the op-amp output by the op-amp noise current is again found by noting that the minus input is held equal to the plus input via op-amp feedback. The only way for that to hold is for I_{nop} to flow totally through Z_{for} and not through Z_{back}.

$$|V_{nopInop}|^2 = I_{nop}^2 |Z_{for}|^2 \quad (26)$$

For the sake of convenience, we may count the noise of R_4 as part of the op-amp output. We now have all of the noise terms at the op-amp output.

$$V_{noptot}^2 = V_{nopR3}^2 + V_{nopp}^2 + V_{nopR2}^2 + V_{nopInop}^2 + V_{nR4}^2 \quad (27)$$

This noise needs merely to be passed through a now noiseless R_4/C_4 filter function to provide the noise from the active filter to be presented to the steering input of the VCO.

$$V_{nout}^2 = \frac{V_{noptot}^2}{\omega^2 C_4^2 R_4^2 + 1} \quad (28)$$

With the above, we have all of the major open-loop noises that will be shaped by the loop except for what is sometimes called the "PLL noise." This is something of a misleading term since it typically means just the noise of the charge pump and dividers in the PLL—not the total PLL noise. It's not given above in the noise sources review, since it's normally specified as a closed-loop noise after shaping. It will be covered in Part 3 along with the methods for analyzing how noise is shaped, optimum loop bandwidth, and the synthesizer IC figure of merit for evaluating chip noise. mw

REFERENCES

1. "Design Methods of Modern Ultra-Low Noise Synthesizers," *Microwaves & RF,* Farron Dacus, Dec. 2018.

2. *Microwave and Wireless Synthesizers, Theory and Design*, Ulrich L. Rohde, John Wiley and Sons, 1997.

3. *Low Noise Electronic System Design,* C.D. Motchenbacher and J.A. Connelly, John Wiley, 1993. This outstanding classic work is highly recommended.

4. *PLL Performance, Simulation, and Design,* Dean Banerjee, first edition 1998. The 5th edition of this outstanding reference, published in 2017, may be freely downloaded at: *http://www.ti.com/tool/pll_book.*